Implications of Electroweak Precision Tests*†

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" In any event, it is always a good idea to try to see how much or how little of our theoretical knowledge actually goes into the analysis of those situations which have been experimentally checked."

R.P. Feynman [1]

The spirit of this talk is best charcterized by the above quotation.

Tree-level Predictions

In this spirit, let us look at the implications of electroweak precision data from LEP and the $W^\pm$ mass. The quality of these data is best appreciated by starting from the tree-level predictions. From the input of

$$\begin{aligned} \alpha(0)^{-1} &= 137.0359895(61) \\ G_\mu &= 1.16639(2) \cdot 10^{-5}\mathrm{GeV}^{-2} \\ M_Z &= 91.1899 \pm 0.0044\mathrm{GeV} \end{aligned}$$

one may predict the partial width of the Z^0 for decay into leptons, Γ_l, the weak mixing angle, $\bar{s}^2_W$, and the mass ratio, $M_{W^\pm}/M_Z$. A comparison of these data with the tree-level predictions shows that the simple $\alpha(0)$ tree-level prediction fails by several standard deviations. The $\alpha(0)$ tree-level prediction yields,

$$\begin{aligned} \bar{s}^2_W(th) &= 0.2121, \\ \Gamma_l(th) &= 84.85 \text{ MeV}, \\ \frac{M_{W^\pm}(th)}{M_Z} &= 0.8876, \end{aligned}$$

which is to be compared with the experimental data [2, 3]

$$\begin{aligned} \bar{s}^2_W(\text{all asymm. LEP}) &= 0.23223 \pm 0.00050 \\ \bar{s}^2_W(\text{all asymm. LEP } + \text{ SLD}) &= 0.23158 \pm 0.00045 \\ \Gamma_l &= 83.98 \pm 0.18 \text{ MeV} \\ \frac{M_{W^\pm}}{M_{Z^0}} &= 0.8814 \pm 0.0021 \end{aligned}$$

Loop-Effects

Concerning loop effects, I follow the 1988 stategy of Gounaris and myself, "to isolate and to test directly the "new physics" of boson loops and other new phenomena by comparing with and looking for deviations from the predictions of the dominant-fermion-loop results"[4], i.e., let us discriminate between fermion-loop vacuum-polarization contributions to photon propagation *as well as Z^0 and $W^\pm$ propagation* on the one hand and boson-loop effects on the other hand. The reason for such a distinction is in fact obvious: the fermion-loop effects can be precisely predicted from the known couplings of the leptons and (light) quarks, while the other loop effects, e.g. vacuum-polarization involving boson pairs and vertex corrections, depend on empirically unknown couplings among the vector bosons (including the Higgs scalar boson in the case of bosonic vacuum-polarization diagrams). In fact, it is the difference between the

fermion-loop results and the full one-loop results which sets the scale for the precision needed for tests of the theory of electroweak interactions beyond (trivial) fermion-loop effects. One should remind oneself that the experimentally unknown bosonic interaction properties are right at the heart of renormalizability of the electroweak theory. The necessary precision for such tests of the theory beyond the leading fermionic contributions has only been reached by the data presented this year (Moriond '94 [2] and Glasgow conference [3])

In our analysis [5], we restrict ourselves to the leptonic observables. The extension to hadronic decays is formulated in [6].

In figs. 1 to 3 from [5], we show the above-mentioned experimental data compared with various theoretical predictions:

i) The $\alpha(M_Z^2)$ tree-level prediction, which is obtained by taking into account the change in the electromagnetic coupling due to leptons and quarks between the low energy scale of $\alpha(0)$ and the scale M_Z^2 by the replacement [7]

$$\alpha(0)^{-1} \rightarrow \alpha(M_Z^2)^{-1} = 128.87 \pm 0.12$$

in the tree-level formulae. It is represented by the isolated point in figs. 1 to 3.

ii) The fermion-loop prediction, which takes into account the quark- and lepton-loop contributions not only to the photon propagator but also to the Z^0 and the $W^\pm$ propagator (the latter one entering the theoretical predictions via G_μ and the top-quark loop). In figs. 1 to 3 the result is indicated by the lines with square insertions, marking the assumed mass of the top quark.

iii) The full one-loop standard model result, which includes all effects due to vacuum polarization, vertex- and box contributions and consequently depends on trilinear and quadrilinear couplings of the bosons among each other and the mass, m_H, of the Higgs boson.

We conclude that

- contributions beyond the $\alpha(M_Z^2)$ tree-level prediction, i.e., electroweak corrections (in addition to the purely electromagnetic ones entering the running of $\alpha(0)$ to $\alpha(M_Z^2)$) are surely needed (a point also stressed by Okun and collaborators [8]),
- contributions beyond *the full fermion-loop* results are necessary,
- there is agreement with the full one-loop result of the $SU(2)\times U(1)$ theory which provides bosonic loop corrections in addition to the fermion loops.

The question immediately arises what can be said about the nature of the bosonic loops which lead to the final agreement between theory and experiment in figs. 1 to 3.

Effective Lagrangian, $\Delta x, \Delta y, \epsilon$ Parameters

This question can best be answered by an analysis in terms of the parameters $\Delta x, \Delta y$ and ϵ which within the framework of an effective Lagrangian [5] specify various possible sources of $SU(2)$ violation. The parameter x is related to $SU(2)$ violation in the triplet of charged and neutral (unmixed) vector boson via

$$M_{W^\pm}^2 = (1+\Delta x) M_{W^0}^2 \equiv x M_{W^0}^2,$$

while Δy specifies $SU(2)$ violation among the $W^\pm$ and W^0 couplings to fermions,

$$g^2_{W^\pm}(0) \equiv M^2_{W^\pm} 4\sqrt{2} G_\mu = (1+\Delta y) g^2_{W^0}(M^2_Z) \equiv y g^2_{W^0}.$$

Finally, the parameter ϵ refers to a mixing strength, when formulating the theory in terms of current mixing á la Hung Sakurai [9],

$$\mathcal{L}_{\rm mix} \equiv \frac{e(M^2_Z)}{g_{W^0}(M^2_Z)} (1-\epsilon) A_{\mu\nu} W^{\mu\nu}_0.$$

Describing electroweak interactions of leptons at the Z^0 in terms of the mentioned effective Lagrangian incorporating the three possible sources of $SU(2)$ violation, one predicts for the observables $\bar{s}^2_W, M_{W^\pm}$ and Γ_l,

$$\begin{aligned}
\bar{s}^2_W(1-\bar{s}^2_W) &= \frac{\pi\alpha(M^2)}{\sqrt{2} G_\mu M^2_Z} \frac{y}{x} (1-\epsilon) \frac{1}{\left(1 + \frac{\bar{s}^2_W}{1-\bar{s}^2_W}\epsilon\right)}, \\
\frac{M^2_{W^\pm}}{M^2_Z} &= (1-\bar{s}^2_W) x \left(1 + \frac{\bar{s}^2_W}{1-\bar{s}^2_W}\epsilon\right), \\
\Gamma_l &= \frac{G_\mu M^3_Z}{24\pi\sqrt{2}} \left(1 + (1-4\bar{s}^2_W)^2\right) \frac{x}{y} \left(1 - \frac{3\alpha}{4\pi}\right).
\end{aligned}$$

For $x = y = 1$ (i.e., $\Delta x = \Delta y = 0$) and $\epsilon = 0$ one recovers the $\alpha(M^2_Z)$ tree-level results mentioned previously. For later usage, we introduce the mixing angle s^2_0 via

$$s^2_0(1-s^2_0) \equiv \frac{\pi\alpha(M^2_Z)}{\sqrt{2} G_\mu M^2_Z}.$$

By inverting the above relations, $\Delta x, \Delta y$ and ϵ may now be deduced from the experimental data on $\bar{s}^2_W, \Gamma_l$ and $M_{W^\pm}$. On the other hand, $\Delta x, \Delta y$ and ϵ may be theoretically determined in the standard electroweak theory at the one-loop level, always strictly discriminating between pure fermion-loop predictions and the rest which contains the unknown bosonic couplings. The striking results of such an analysis are shown in figs. 4, 5, 6.

According to fig. 4, the data in the $(\epsilon, \Delta x)$ plane are well described if Δx and ϵ are approximated by their fermion-loop values,

$$\begin{aligned}
\Delta x &= \Delta x_{\rm ferm}(\alpha(M^2_Z), s^2_0, m^2_t) + \Delta x_{\rm bos}(\alpha(M^2_Z), s^2_0, \ln m^2_H) \\
&\cong \Delta x_{\rm ferm}(\alpha(M^2_Z), s^2_0, m^2_t), \\
\epsilon &= \epsilon_{\rm ferm}(\alpha(M^2_Z), s^2_0, m^2_t) + \epsilon_{\rm bos}(\alpha(M^2_Z), s^2_0, \ln m^2_H) \\
&\cong \epsilon_{\rm ferm}(\alpha(M^2_Z), s^2_0, m^2_t).
\end{aligned}$$

The logarithmic dependence on the Higgs mass, m_H, and the small contributions of $\Delta x_{\rm bos}$ and $\epsilon_{\rm bos}$ imply the well-known result that the data are very insensitive to the mass of the Higgs scalar. Values between 60 GeV and more than 1 TeV can easily be accomodated [10].

In contrast, a striking effect appears in figs. 5 and 6. The theoretical predictions are clearly inconsistent with the data, unless the fermion-loop contributions to Δy (denoted by lines with

small squares in figs. 4 to 6) are supplemented by an additional term, which in the standard electroweak theory contains bosonic effects,

$$\Delta y = \Delta y_{\text{ferm}}(\alpha(M_Z^2), s_0^2, \ln m_t) + \Delta y_{\text{bos}}(\alpha(M_Z^2), s_0^2).$$

Remembering that Δy by definition relates the $W^\pm$ coupling measured in $\mu^\pm$ decay to the (unmixed) Z^0 coupling,

$$g^2_{W^\pm}(0) = (1 + \Delta y) g^2_{W^0}(M_Z^2),$$

it is not surprising that Δy_{bos} contains vertex and box corrections originating from $\mu^\pm$ decay as well as vertex corrections at the $Z^0 f\bar{f}$ vertex. While Δy_{bos} obviously depends on the trilinear couplings among the vector bosons, it is independent of the Higgs mass, m_H. (Note that Δy_{ferm} and Δy_{bos} are separately unique and gauge-invariant quantities in the $SU(2) \times U(1)$ theory.)

In conclusion, the experimental data have become accurate enough to be *sensitive to loop effects which are independent of m_H but depend on the self-interactions of the vector bosons, in particular on the trilinear couplings entering the $W^\pm f\bar{f}'$ and $Z^0 f\bar{f}$ vertex corrections.*

Electroweak Interactions in Higgs-less Massive Vector Boson Theory.

As the experimental results for Δx and ϵ are well represented by neglecting all effects with the exception of fermion loops, and as the bosonic contribution to Δy which is seen in the data is independent of m_H, the question as to the role of the Higgs mass and the concept of the Higgs mechanism with respect to precision tests immediately arises.

More specifically, one may ask the question whether the experimental results, i.e. $\Delta x, \Delta y, \epsilon$, can be predicted even without the very concept of the Higgs mechanism.

In [11] we start from the well-known fact that the standard electroweak theory without Higgs particle can credibly be reconstructed within the framework of a massive vector-boson theory with the most general mass mixing term which preserves electromagnetic gauge invariance. This theory is then cast into a form which is invariant under local $SU(2) \times U(1)$ transformations by introducing three auxiliary scalar fields á la Stueckelberg. As a consequence, loop calculations may be carried out in an arbitrary R_ξ gauge.

Explicit loop calculations show that indeed the Higgs-less observable Δy, evaluated in the massive vector-boson theory (MVB), coincides with Δy evaluated in the standard electroweak theory, i.e. in particular for the bosonic part, we have[1]

$$\Delta y_{\text{bos}}^{\text{MVB}} \equiv \Delta y_{\text{bos}}^{\text{St.M.}}.$$

As for Δx_{bos} and ϵ_{bos}, one finds that the massive-vector-boson theory and the standard model differ by the replacement $\ln m_H \Leftrightarrow \ln \Lambda$, where Λ denotes an ultraviolet cut-off. For $\Lambda \leq 1$ TeV, accordingly,

$$\begin{aligned} \Delta x^{\text{MVB}} &\cong \Delta x_{\text{ferm}}^{\text{MVB}} = \Delta x_{\text{ferm}}^{\text{St.M.}}, \\ \epsilon^{\text{MVB}} &\cong \epsilon_{\text{ferm}}^{\text{MVB}} = \epsilon_{\text{ferm}}^{\text{St.M.}}. \end{aligned}$$

In conclusion, the massive-vector-boson theory can indeed be evaluated at one-loop level at the expense of introducing a logarithmic cut-off, Λ. This cut-off only affects Δx and ϵ, whose bosonic contributions cannot be resolved experimentally.

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[1]Actually, in the standard theory there is an additional term which depends on the Higgs mass like $1/m_H^2$ and is irrelevant numerically for $m_H \geq 130$ GeV.

The quantity Δy, whose bosonic contributions are essential for agreement with experiment, is independent of the Higgs mechanism. It depends on the trilinear couplings of the vector bosons among each other, which enter the vertex corrections at the $W^\pm$ and Z^0 vertices. Even though the data cannot discriminate between the massive vector-boson theory and the standard model with Higgs scalar, the Higgs mechanism yields nevertheless the only known simple physical realization of the cut-off Λ (by m_H) which guarantees renormalizability.

Conclusions

- The analysis of the Z^0 data and the $W^\pm$ mass in terms of an effective Lagrangian with $SU(2)$ beaking via $\Delta x, \Delta y$ and ϵ yields for these parameters values which are of the order of magnitude of radiative corrections. This in itself consitutes a major triumph of the $SU(2)_L \times U(1)_Y$ symmetry principle which is at the root of present-day electroweak theory.
- The data have reached such a high precision that contributions to the parameter Δy are needed beyond the ones induced by (vacuum polarization) fermion loops to the photon, Z^0 and $W^\pm$ propagators. These contributions are connected with vertex corrections at the $W^\pm f\bar{f}'$ and $Z^0 f\bar{f}$ vertices which contain truely non-Abelian (trilinear) couplings among the vector bosons.
- The parameters Δx and ϵ, consistently reproduce the data (for $m_t \simeq 175$ GeV), if approximated by fermion loops, $\Delta x \cong \Delta x_{\rm ferm}$ and $\epsilon \approx \epsilon_{\rm ferm}$.
- The data by themselves do not discriminate a massive-vector-boson theory from the standard theory based on the Higgs mechanism. The issue of mass generation will remain open until the Higgs scalar will be found - or something else?

Acknowledgement

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References

[1] R.P. Feynman in "Theory of Fundamental Processes", New York, Benjamin 1962, p. VII

[2] "La Thuile and Moriond Conferences, March 1994

[3] Glasgow International Conference on High Energy Physics, July 1994

[4] G. Gounaris and D. Schildknecht, Z. Phys. C40 (1988) 447, Z. Phys. C42 (1989) 107

[5] M. Bilenky, K. Kolodziej, M. Kuroda, D. Schildknecht, Phys. Lett. B319 (1993) 319
S. Dittmaier, K. Kolodziej, M. Kuroda, D. Schildknecht, Nucl. Phys. B426 (1994) 249

[6] S. Dittmaier, M. Kuroda, D. Schildknecht, Bielefeld priprint BI-TP 94/62 to be submitted to Nucl. Phys. B

[7] H. Burkhardt, F. Jegerlehner, G. Penso and C. Verzegnasse, Z. Phys. C43 (1989) 497

[8] V. Novikov, L. Okun, M. Vysotsky, A. Rozanov, CERN-TH 7217 (1994), Mod. Phys. Lett. A, to be published

[9] P.Q. Hung and J.J. Sakurai, Nucl. Phys. B143 (1978) 81

[10] e.g., G. Montagna, O.Nicrosini, G. Passarino, F. Piccinini, Phys. Lett. B335, (1994) 484

[11] S. Dittmaier, C. Grosse-Knetter, D. Schildknecht, BI-TP 94/31, hep-ph/9406378, to appear in Z. Phys. C

Figure 1:

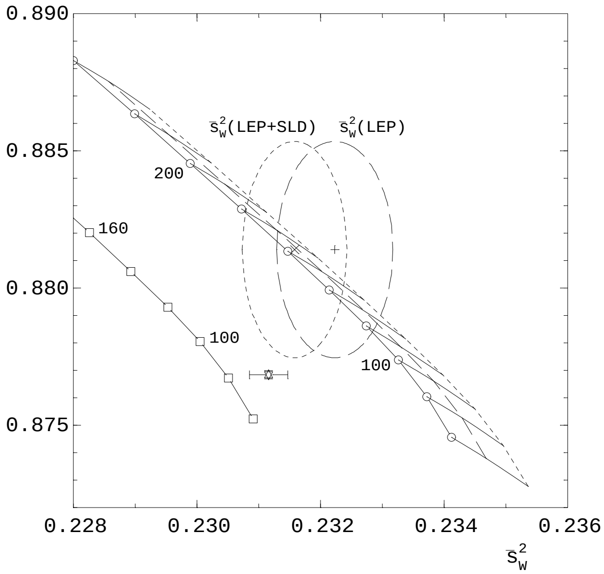


Figure 2:

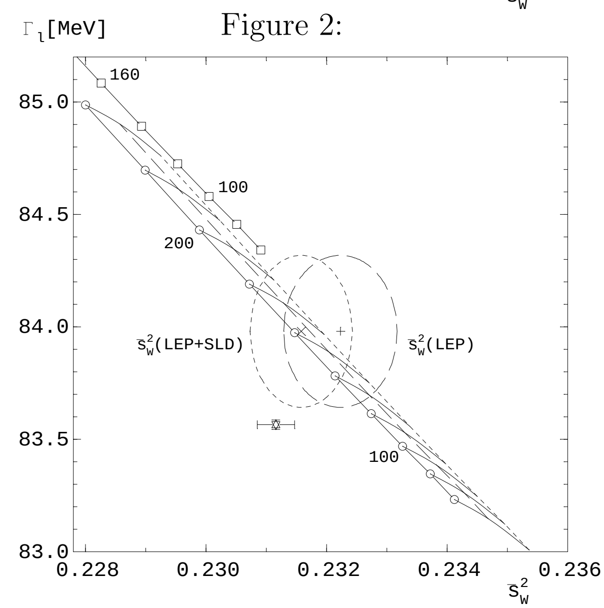
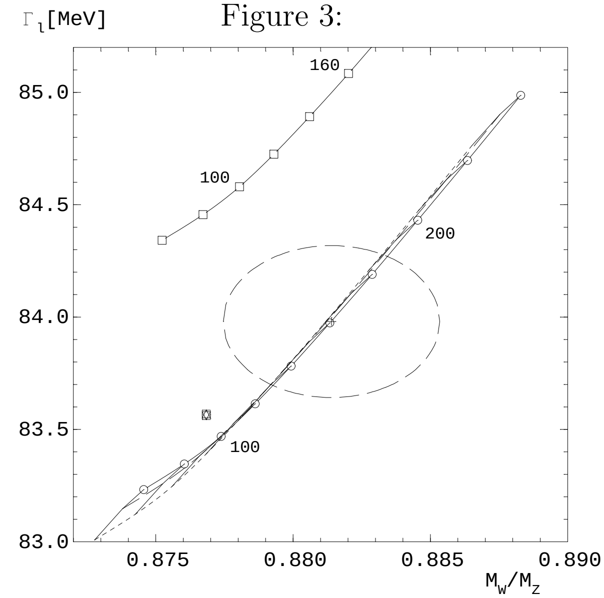


Figure 3:



Figs. 1,2,3: The experimental data on $(M_W/M_Z,\, \bar{s}^2_W, \Gamma_l)$ compared with theory.

Figure 4:

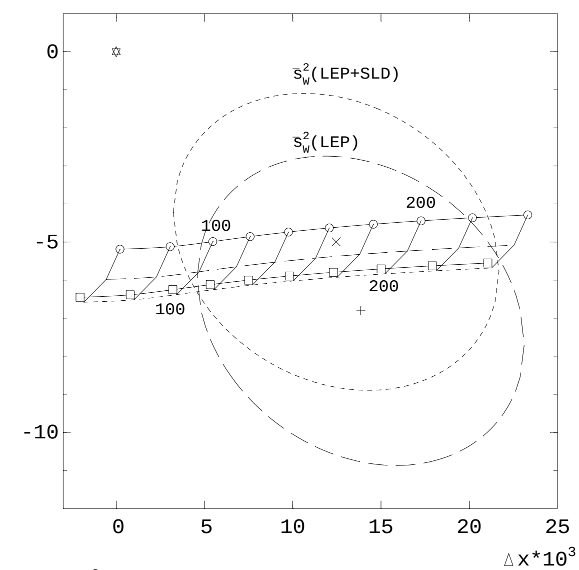


Figure 5:

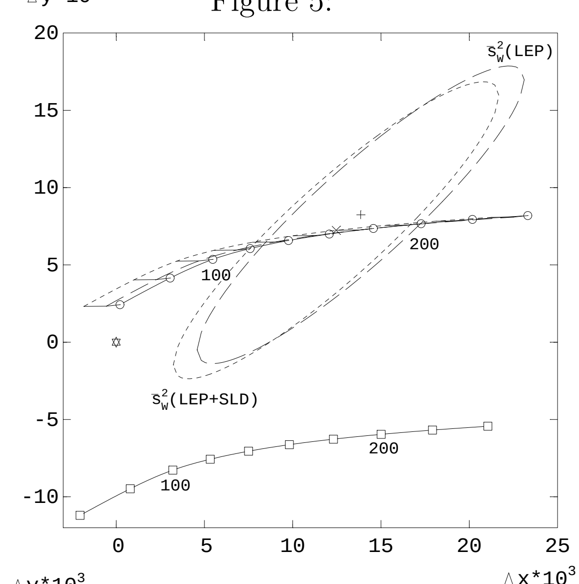
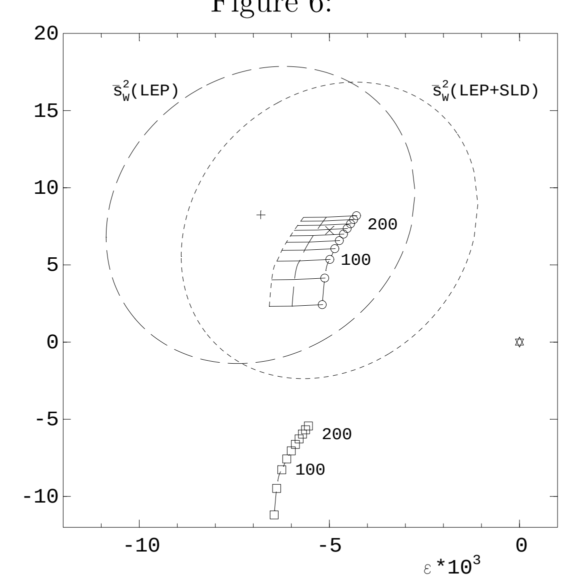


Figure 6:



Figs. 4,5,6: The experimental data on $\Delta x, \Delta y, \epsilon$ compared with theory.